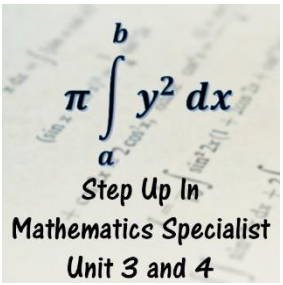
5.1 Implicit Differentiation and the Incremental Formula

Problems Worksheet



1. Differentiate the following terms with respect to x.

   a. $3x^2$

   b. $3y^2$

   c. xy

   d. x^2y

   e. xy^2

   f. $\sin xy$

   g. $(2x - 2xy)^3$

   h. $\sqrt{xy}$

2. Determine $\frac{dy}{dx}$ in terms of x and y without first rearranging the equation.

   a. $x^2 + y^2 = 25$

   b. $xy + \cos x = 3y$

   c. $xy + \cos xy = 3y$

3. Determine the coordinates of all points on the following curves with the gradient specified.

   a. $x = 0.1y^2 - y + 4$, gradient 1.

   b. $x^2 - y^2 = 6$, gradient 2.

4. Determine $\frac{d^2y}{dx^2}$ in terms of x and y.

   a. $y^2 + 2xy = x^2$

   b. $e^x = xy$

5. Use an incremental formula approach to estimate the following.

    a. The increase in surface area of a cube when its length increases from $5\ cm$ to $5.1\ cm$.

    b. The increase in volume of a snowball when its radius increases from $4\ cm$ to $4.1\ cm$.

    c. The increase in area of an equilateral triangle when its length increases from $5\ cm$ to $5.1\ cm$.

    d. The decrease in area of a regular hexagon when its side length reduces from $4\ cm$ to $3.9\ cm$.

6. A small manufacturer determines that the total cost (in dollars) of producing n units of their product each day is given by $C(n) = 1000 + 20n + 40\sqrt{n}$ and each unit is sold for \$40.

    a. Determine an expression for the profit earned selling all n items produced per day.

    b. Calculate how many items must be produced and sold for the business to "break even" for the day. Assume that all products manufactured are also sold.

    c. Determine the values of $C'(70)$ and $P'(70)$ and state their meaning.